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MATHEMATICAL AND NUMERICAL MODEL FOR BLOOD FLOW IN LARGE VESSELS WITH SOME PATHOLOGY

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ABSTRACT: Using a non-Newtonian mathematical model for the blood flow in large vessels – elaborated and presented already by us in some previous papers we make some remarks on the wall shear stress (*WSS*) in the case of a human abdominal aortic aneurysm (*AAA*). We focused on the mechanical conditions which would lead to the rupture of the vascular vessel with aneurysm.

The original feature is that the established condition is not backed on the real blood flow but only on the data got via the above mentioned model. This leads to a more suitable tool of risk rupture prediction which avoids almost completely the investigation in vivo.

Keywords: wall shear stress, aneurysm, stress tensor, non-Newtonian model, rupture risk,

INTRODUCTION

In this research the blood is considered a non-Newtonian fluid, with variable coefficient of viscosity, under an unsteady (pulsatile) flow regime connected with the rhythmic pumping of the blood by the heart.

We also admit the incompressibility and homogeneity of the blood while the exterior body forces (as gravity) are neglected. Concerning the limiting walls of the vessels, we accept their viscoelastic behavior, the whole configuration having an axial symmetric geometry versus the vertical axis *Oz*.

According to this model the flow equations result from the general Cauchy equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = div\mathbf{T},$$

where the stress tensor $\mathbf{T} = -p\mathbf{I} + 2(\mu_s + \mu_{RBC})\mathbf{D}$, $\mathbf{I}$ being the unit tensor, the scalar p is the physical pressure, $\mathbf{D}$ is the rate of strain tensor, μ_s representing the (constant) plasma viscosity while μ_{RBC} is given by the Cross model, i.e.,

$$\mu_{RBC} = \frac{\mu_0^*}{1+(k\dot{\gamma})^{1-n}},$$

with $\dot{\gamma}$ the shear rate, μ_0^* the viscosity coefficient of the blood, k a time constant and n the index for a shear thinning behavior.

The above evolution equations are joined to some boundary conditions which express the existence of a pressure gradient along *Oz* axis according to the heart beats and implicitly to the rhythmic blood pushing into the vessel, a feature which is important in approaching the large vessels. Thus we have:

$$\frac{\partial v}{\partial r} = 0 \text{ and } u = 0 \text{ for } r = 0,$$

while at $r = R$, due to the viscoelastic behavior of the vessel's wall, the velocity of the blood must be equal to the "displacement" velocity of the wall. The boundary conditions at "edges" $z = 0$ and $z = L$ of the vessel, agree with the physiological pulse velocity given by a periodic time-varying function .

To describe the viscoelastic behavior of the vessel's wall we have used the generalized Maxwell model, which is the most general form of the linear model for viscoelasticity ([4], [5]).

Evaluation of the stress vector $\vec{\mathbf{T}}$ in the points of the aneurysm boundary

Mostly, we intend to set up a mechanical condition whose fulfillment would lead with a high probability to the rupture of the aneurysm and consequently to the damage of the vessel wall of an *AAA*. We will focus on the case of a human abdominal aortic with a double aneurysm(AAA) considered by Finol et. al. [6].

The above mentioned rupture takes place when the *WSS* evaluated on the boundary of aneurysm overpasses the internal cohesion forces assessed on the same boundary of the aneurysm. But these internal cohesion forces are connected with the projection of the stress vector $\vec{T}$ (Maxwell model) on the unit tangent to the boundary vector $\vec{t}$.

We accept, in a plane $\varphi = const$, that the equation of the vessel wall (with aneurysm) could be expressed in a Cartesian coordinate system formed by the radial axis r and the axis of symmetry z by the equation $z = z(r)$.

The expression for the stress vector is $\vec{T} = \mathbf{T}\vec{n}$, where $\mathbf{T}$ is the stress tensor in the Maxwell model while $\vec{n}$ is the normal unit vector at the considered point, namely

$$\vec{n}\left(\frac{-\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}, \frac{1}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}, 0\right).$$

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The components of the stress tensor **T**, using the general Maxwell model for viscoelasticity are $T_{ij} = s_{ij} - pI_{ij}$, where s_{ij} are the components of the stress deviator $\mathbf{s} = 2G_0(\eta_0\mathbf{e} + (\sum_{m=1}^{4}\eta_m q_m)\mathbf{I})$, **e** being the rate of strain deviator ($\mathbf{e} = \boldsymbol{\varepsilon} - 1/3Tr(\boldsymbol{\varepsilon})\mathbf{I}$), $\boldsymbol{\varepsilon}$ being the rate of strain tensor, G_0 the shear modulus, $G_0\eta_0$ the long term shear modulus.

Thus we have for the components of the stress tensor (Maxwell)

$$T_{rr} = 2G_0\left[\eta_0\left(\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p$$

$$T_{rz} = 2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right]$$

$$T_{zr} = 2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right]$$

$$T_{zz} = 2G_0\left[\eta_0\left(\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p,$$

where $A = \sum_{m=1}^{4}\eta_m q_m$, η_m ($m = 0,1,2,3,4$) being coefficients of the relative rigidity of the wall and q_m are parameters attached to the extension of the wall, ($\sum_{m=0}^{4} q_m = 1$), while *p* is the pressure.

We denoted by *u* and *v* the components of the blood velocity $\vec{v}$ on the directions of *r* and *z* respectively. Obviously all the components $T_{\varphi j}$, $j = r, z, \varphi$ are zero and also $\frac{\partial}{\partial \varphi} \equiv 0$ (due to the axial symmetry)

As $\vec{T} = \mathbf{T}\vec{n}$, with $\vec{T}(T_r, T_z)$, where

$T_r = T_{rr}n_r + T_{zr}n_z$ and

$T_z = T_{rz}n_r + T_{zz}n_z$,

while the unit normal vector to the curve $z = z(r)$ ($\varphi = const$) is

$$\vec{n}\left(\frac{-\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}, \frac{1}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}\right).$$

and the unit tangent vector to the same curve (in a plane $\varphi = const$) is

$$\vec{t}\left(\frac{1}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}, \frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}\right)$$

we can write

$$T_r = -\left\{2G_0\left[\eta_0\left(\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}} + 2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right]\frac{1}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}$$

and

$$T_z = -2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right]\frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}} + \left\{2G_0\left[\eta_0\left(\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{1}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}.$$

Then, in the points of the aneurysm $z = z(r)$, the projection of the Maxwell stress vector $\vec{T}$ on the direction of the unit tangent vector to the vessel wall $\vec{t}$ will be

$$\vec{T}\cdot\vec{t} = T_r \cdot \frac{1}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}} + T_z \cdot \frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}$$

, namely

$$\vec{T}\cdot\vec{t} = -\left\{2G_0\left[\eta_0\left(\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{1+\left(\frac{dz}{dr}\right)^2} + 2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right]\frac{1}{1+\left(\frac{dz}{dr}\right)^2} -$$

$$-2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right]\frac{\left(\frac{dz}{dr}\right)^2}{1+\left(\frac{dz}{dr}\right)^2} + \left\{2G_0\left[\eta_0\left(\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{1+\left(\frac{dz}{dr}\right)^2}.$$

This product would estimate the internal cohesion forces in the points of the vessel wall with aneurysm.

Concerning the wall shear stress, observing the conditions of the considered Cross law for blood, we could write:

$$WSS = \left(\mu_s + \frac{\mu_0^*}{1 + k\dot{\gamma}^{1-n}}\right)\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right),$$

where $\dot{\gamma} = \sqrt{2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{u}{r}\right)^2}$ and thus

$$WSS = \left(\mu_s + \frac{\mu_0^*}{1 + k\left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{u}{r}\right)^2\right]^{\frac{1-n}{2}}}\right)\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right).$$

Consequently the "rupture" of the vessel wall would take place if the *WSS* overpasses $\vec{T}\cdot\vec{t}$ (both considered in absolute value)

i.e.,

$$\left|-\left\{2G_0\left[\eta_0\left(\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{1+\left(\frac{dz}{dr}\right)^2} + 2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right]\frac{1}{1+\left(\frac{dz}{dr}\right)^2} - \right.$$

$$\left. -2G_0\left[\eta_0 \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right]\frac{\left(\frac{dz}{dr}\right)^2}{1+\left(\frac{dz}{dr}\right)^2} + \left\{2G_0\left[\eta_0\left(\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{1+\left(\frac{dz}{dr}\right)^2}\right| <$$

$$< \left|\left(\mu_s + \frac{\mu_0^*}{1 + k\left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{u}{r}\right)^2\right]^{\frac{1-n}{2}}}\right)\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right|.$$

In the sequel we will try to rewrite this exact analytical condition under a more convenient form for practical use.

1.* Construction of an analytical approximation of *WSS

In what follows, we intend to build up an analytical expression for approximating the *WSS* considered, at a certain moment, as a function of the *axial coordinate z*. To achieve that, we will use the numerical data of the joined table (table 1) and we interpolate the *WSS* along the whole boundary of the aneurysm.

Denoting then by WSS^i the values $WSS(z_i)$ at a certain fixed moment (where z_i are axial coordinates of the points $i = 1,2,3,4,5$), supposing $WSS \in C^2(z)$, we will introduce the spline interpolation function $S(z) \in C^2(z)$ on the whole boundary.

Table 1.

Values of *WSS* (at a certain time t = 7.7s) at the 5 considered points as shown on figure 1

Points	*z* (cm)	*WSS* (N/m^2)
1	1	-1.25
2	2	0.175
3	3	-2.55
4	5	0.16
5	7	-2.45

This type of approximation made by cubic functions, beside the fact that ensures a minimum curvature of it, could "preserve" the shape of the exact *WSS* and consequently it has the extremum points close to those of the exact *WSS* [8][1]. That is why we are using it, our final goal being to assess the absolute minimum and maximum of *WSS* for anticipating a possible "rupture" of the vessel with aneurysm.

Denoting by $M_i = S''(z_i)$, $i = 0,1,2,3,4,5,6$ (where the point 0 and 6 correspond to the "farfield" of the aneurysm – where the deviation of the *WSS* is practically absent[2] and consequently $WSS^0 = WSS^6 = WSS'^0 = WSS'^6 = 0$),

if $h_i = z_i - z_{i-1}$, the spline function joined to the "*i*" subinterval, is given by [9]

$$S_i(z) = \frac{M_i(z - z_{i-1})^3 + M_{i-1}(z_i - z)^3}{6h_i} + \left(WSS^{i-1} - \frac{M_{i-1}h_i^2}{6}\right)\frac{z_i - z}{h_i} + \left(WSS^i - \frac{M_i h_i^2}{6}\right)\frac{z - z_{i-1}}{h_i}$$

$i = \overline{1,6}$

By imposing the additional condition $M_0 = M_6 = 0$ (what is in complete accord with the absence of the aneurysm "at farfield"[1]), we have also assured the uniqueness of this spline interpolation function $S(z)$ [8].

Concerning the constants M_i they can be obtained by solving the following algebraic linear system [9]

$$a_0 M_0 + c_0 M_1 = d_0$$

$$b_i M_{i-1} + a_i M_i + c_i M_{i+1} = d_i,\ i = 1,2,3,4,5$$

$$b_6 M_5 + a_6 M_6 = d_6,$$

where $b_i = \dfrac{h_i}{h_i + h_{i+1}}$, $c_i = 1 - b_i$,

$$d_i = \frac{6}{h_i + h_{i+1}}\left(\frac{WSS^{i+1} - WSS^i}{h_{i+1}} - \frac{WSS^i - WSS^{i-1}}{h_i}\right),$$

$i = 1,2,3,4,5$;

$b_6 = 1$, $c_0 = 1$, $d_0 = \dfrac{6}{h_1}\left(\dfrac{WSS^1 - WSS^0}{h_1} - WSS'^0\right)$,

$d_6 = \dfrac{6}{h_6}\left(WSS'^6 - \dfrac{WSS^6 - WSS^5}{h_6}\right)$, $a_i = 2$,

$i = 0,1,2,3,4,5,6$.

Concerning the error of the approximation it is of the same order as of the certain powers of $h = \max\limits_i (z_i - z_{i-1})$, the degree of accuracy increasing together with the regularity of *WSS* [9].

If we want to calculate the critical points of the spline approximation for *WSS*, from

$$\frac{dS_i}{dz} = \frac{z^2(M_i + M_{i-1}) - 2z(z_{i-1}M_i + z_i M_{i-1}) + M_i z_{i-1}^2 + M_{i-1}z_i^2 + 2(WSS^i - WSS^{i-1}) - 2/3h_i^3(M_i - M_{i-1})}{2h_i}$$

we have to impose the condition

$$\Delta_i = (z_{i-1}M_i + z_i M_{i-1})^2 - (M_i + M_{i-1})\left[M_i z_{i-1}^2 + M_{i-1}z_i^2 + 2(WSS^i - WSS^{i-1}) - 2/3h_i^3(M_i - M_{i-1})\right] \geq 0$$

The critical points, for each subinterval (z_{i-1}, z_i), are given by

$$z_{1,2}^i = \frac{(z_{i-1}M_i + z_i M_{i-1}) \pm \sqrt{\Delta_i}}{(M_i + M_{i-1})}.$$

The relative extremum values of this approximation (and implicitly of *WSS*) should be found among the values of

$S_i(z_{1,2}^i)$, $i = 1,2,3,4,5,6$,

where we must consider only those $z_{1,2}^i \in (z_{i-1}, z_i)$ (at least approximately).

Then by considering $\min\limits_i \left|S_i(z_{1,2}^i)\right|$ this should be the limit value of *WSS* which once overpassed by the internal cohesion forces evaluated on the aneurysm boundary ($\vec{T} \cdot \vec{t}$) the rupture takes place. Of course this represents a global condition not a local one.

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[1] For improving the shape preservation, instead of a cubic spline interpolant it can be used also the variant of the *pchip* interpolant [8].

[2] Instead of the genuine complete *WSS* we will work with its "deviation" versus the normal artery (without aneurysm) and consequently the fulfillment of the required conditions is assured.

Concerning the projection of the stress vector ($\vec{T}$) on the boundary tangent ($\vec{t}$) using the Maxwell viscoelastic behavior for the blood walls (aneurysm included) it can be obtained via COMSOL 3.3 [3].

In our real clinical study a human abdominal aortic with a double aneurysm(AAA), as it can be found in the paper elaborate by Finol a.o. [6], the linear algebraic system was solved by QuickMath and the solutions are $M_0 = -12.447$, $M_1 = 11.395$, $M_2 = -11.085$, $M_3 = 8.045$, $M_4 = -6.352$, $M_5 = 9.383$, $M_6 = -15.067$.

Immediately we get for the critical points the coordinates (keeping those whose values are close to the inside of the considered subinterval (z_{i-1}, z_i)) $z_1 = 1.402$, $S_1(z_1) = -1.29$; $z_2 = 2.305$, $S_2(z_2) = -1.55$; $z_3 = 3.732$, $S_3(z_3) = -1.59$; $z_5 = 8.25$, $S_5(z_5) = 9.984$; $z_6 = 8.244$, $S_6(z_6) = -0.503$. Concerning $\min_i \left| S_i(z_{1,2}^i) \right|$ it is equal to 0.503.

This should be compared to the maximum value of $\left| \vec{T} \cdot \vec{t} \right|$ evaluated on the boundary. As this maximum value of the internal cohesion forces (got via COMSOL Multiphysics) is 6.175, we may state that at the considered moment t = 7.7s there is no global rupture risk.

Of course these steps must be repeated at all the moments, but an appropriate soft could solve this feature without any special shortcomings.

Numerical simulation

A numerical simulation for this particular situation has been already made by us using COMSOL Multiphysiscs 3.3 ([3]). To avoid the transient effect of the initial conditions the final results of our analysis are presented only for the last 5 periods, although the time integration interval was $t \in [0,10s]$.Choosing 5 particular points on the vessel wall of the *AAA* and evaluating the *WSS* at these points, it has been shown that *WSS* reaches its maximum value in a point located between the two aneurysms (marked with a "red point") and at the exit point of the second one (marked with an "orange point") as it can be seen on figure 1. The values of the *WSS* (through 5 seconds) in these 5 points are given in figures 2-6.

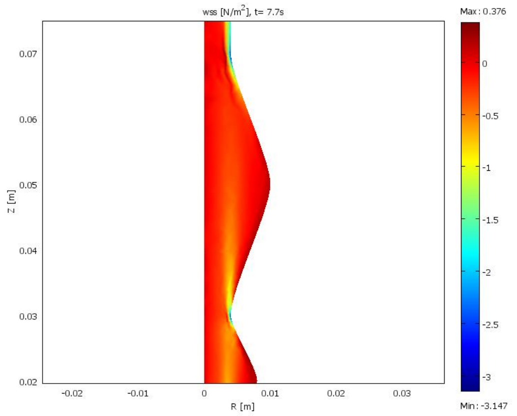

Fig. 1 Surface distribution of the *WSS* in the case of considered *AAA*

At t = 7.7*s*, the maximum value of the *WSS* – at the "red point" is around 2.55, respectively 2.45 at the "orange point". The "-" sign shows that the wall shear stress acts in opposite direction to that of the blood flow.

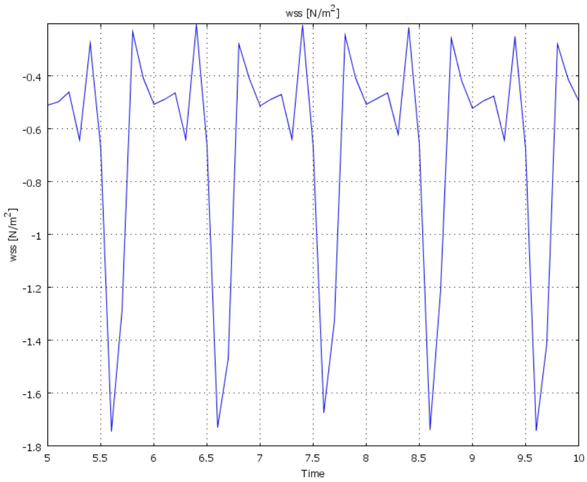

Fig. 2 Variation (5 periods) of wall shear stress at the entry point of the aneurysm with the smaller diameter (green point on figure 1)

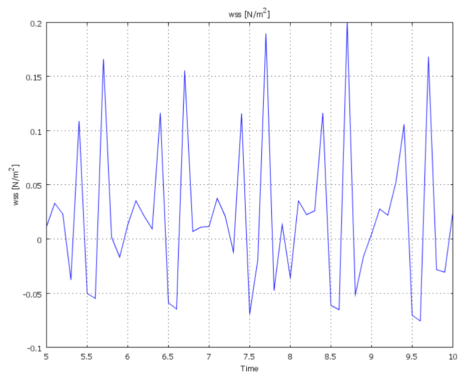

Fig. 3 Variation (5 periods) of wall shear stress in the middle of the aneurysm with the smaller diameter (purple point on figure 1)

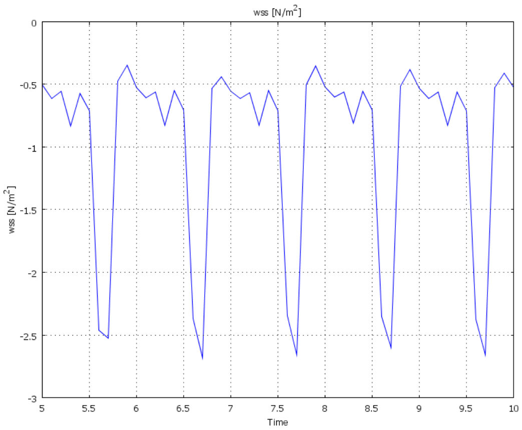

Fig. 4 Variation (5 periods) of wall shear stress at the exit of the aneurysm with the smaller diameter (red point on figure 1)

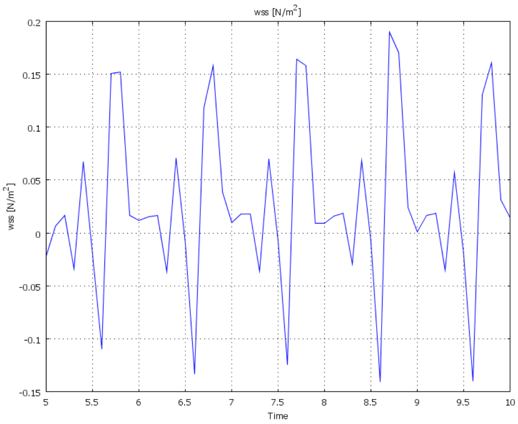

Fig. 5 Variation (5 periods) of wall shear stress in the middle of the aneurysm with the larger diameter (blue point on figure 1)

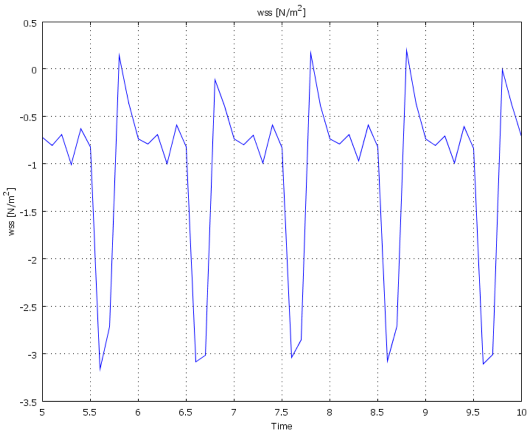

Fig. 6. Variation (5 periods) of wall shear stress at the exit of the aneurysm with the larger diameter (orange point on figure 1)

According to Papaioannou & Stefanos [7] the "normal" value of the *WSS* in the case of the arteries is around 1 N/m^2.

We remark that the values of *WSS* got through our numerical simulations and experiments in the previous research [3] are in total accord with those presented in the work of Finol et al. [6]. This fact validates the accuracy of the use of the Cross type non-Newtonian model for the blood flow together with the viscoelastic behavior (Maxwell) of the vessel walls.

CONCLUSIONS

We remark that this approach leads to a global rupture risk prediction. In our particular clinical case, in spite of the fact that *WSS* overpasses the internal cohesion forces at the point 5 (see table 2, where a rupture risk really exists) we may state that, globally speaking, there is no rupture risk for the considered artery.

Table 2. *WSS* evolution in time versus the corresponding internal cohesion forces at point 5

t (s)	7	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8
WSS (N/m^2)	-0.53	-0.58	-0.53	-0.77	-0.52	-0.66	-2.07	-2.94	-0.48	-0.39	-0.51
$\vec{T}\cdot\vec{t}$ (N/m^2)	-1.12	-0.65	-3.37	-3.37	5.41	-9.36	-14.9	2.72	-1.19	-0.68	-0.77

We also remark that at point 4, where the diameter of the aneurysm is the greatest there is no risk of rupture, what implies the conclusion that the diameter of the aneurysm is not the essential parameter for the evaluation of the rupture-risk.

In the future we intend to try to set up a medical soft for getting the global condition of this rupture-risk.

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